Goethe-Center for Scientific Computing (G-CSC)
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Modeling and Simulation I

(Practical SIM1, WS 2017/18)

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Exercise sheet 5 (Due: Mo., 11.12.2017, 10h)

Exercise 1 (8P + 2P + 2P)
In `Sheet02, Exercise 3` we solved the two-body system, consisting of the earth and an artificial satellite. Now, with far more capable numerical solvers, we can finally solve a much more realistic N-Body system, such as our own solar system. Analogous to `Sheet02, Exercise 3`, the force between two bodies $body_i$ and $body_j$ is defined as:

$$F_{ij} = Gm_i m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{\|\mathbf{r}_j - \mathbf{r}_i\|^3} \tag{1}$$

where G is the gravitational constant, $\mathbf{r}_i$ is defined as the location of $body_i$ and $\|\mathbf{r}_j - \mathbf{r}_i\|$ is the magnitude of the distance between $\mathbf{r}_i$ and $\mathbf{r}_j$. The system of ODEs that describes the behavior of n bodies in space is defined as follows:

$$\frac{d^2\mathbf{r}_1}{dt^2} = \sum_{\substack{j=1 \\ j\neq 1}}^{n} Gm_j \frac{\mathbf{r}_j - \mathbf{r}_1}{\|\mathbf{r}_j - \mathbf{r}_1\|^3}$$

$$\vdots$$

$$\frac{d^2\mathbf{r}_i}{dt^2} = \sum_{\substack{j=1 \\ j\neq i}}^{n} Gm_j \frac{\mathbf{r}_j - \mathbf{r}_i}{\|\mathbf{r}_j - \mathbf{r}_i\|^3}$$

$$\vdots$$

$$\frac{d^2\mathbf{r}_n}{dt^2} = \sum_{\substack{j=1 \\ j\neq n}}^{n} Gm_j \frac{\mathbf{r}_j - \mathbf{r}_n}{\|\mathbf{r}_j - \mathbf{r}_n\|^3}$$

Our simulation will start at 01.12.2017 and ends exactly 31558149 seconds later which is the duration of one earth orbit. The following bodies shall be simulated:

1. Sun
2. Mecrury
3. Venus
4. Earth
5. Moon (Luna)
6. Mars
7. Jupiter
8. Saturn
9. Uranus
10. Neptune
11. Comet Halley (1P)
12. Eros
13. Churiomov Gerasimenko (67P)

To acquire the initial values (mass, **r** and **v**), visit the NASA HORIZONS webpage (http://ssd.jpl.nasa.gov/horizons.cgi). To get data in the required format, the default settings have to be changed:

Current Settings

Ephemeris Type [change] : **VECTORS**
Target Body [change] : **Mars** [499]
Coordinate Origin [change] : **Solar System Barycenter (SSB)** [500@0]
Time Span [change] : Start=**2017-12-01**, Stop=**2018-11-30**, Step=**1 d**
Table Settings [change] : quantities code=**2**; output units=**KM-S**; labels=**NO**
Display/Output [change] : *default* (formatted HTML)

Necessary changes in table settings:

Table Settings

Select vector table output

Type 2 (state vector {x,y,z,vx,vy,vz})

Optional vector-table settings:

output units :	km & km/s -- units to use for distance and velocity
reference plane :	ecliptic and mean equinox of reference epoch -- reference X-Y plane for vectors
reference system :	ICRF/J2000.0 -- reference frame for vector coordinates
aberrations :	Geometric states (no aberration; instantaneous ephemeris states) -- aberration correction
labels :	☐ -- enable labeling of each vector component
delta-T (TDB-UT) :	☐ -- output time-varying difference between TDB and UT time-scales
CSV format :	☐ -- output vector components in Comma-Separated-Variables (CSV) format
object page :	☑ -- include object information/data page on output

Use Settings Above | Default Optional Settings | Cancel

The settings are listed below:

Ephemeris:

- Ephemeris Type: Vector

Coordinate Origin:

- Coordinate Origin: Solar System Barycenter (SSB) [500@0]

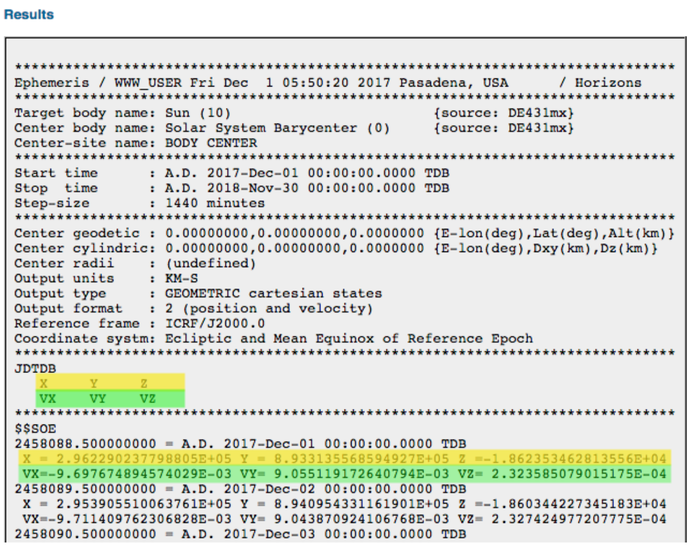
Time Span:

- Time Span: 2017-12-01 to 2018-11-30, 00:00
  Format: (yyyy-mm-dd, HH-MM)

Table Settings:

- Output Units: km & km/s
- Quantities Code: 2 (state vector {x,y,z,vx,vy,vz})

After adjusting the settings, click "Generate Ephemeris". The result is shown as html output, which will look like this:



Results

```
*******************************************************************************
Ephemeris / WWW_USER Fri Dec  1 05:50:20 2017 Pasadena, USA      / Horizons
*******************************************************************************
Target body name: Sun (10)                        {source: DE431mx}
Center body name: Solar System Barycenter (0)     {source: DE431mx}
Center-site name: BODY CENTER
*******************************************************************************
Start time      : A.D. 2017-Dec-01 00:00:00.0000 TDB
Stop  time      : A.D. 2018-Nov-30 00:00:00.0000 TDB
Step-size       : 1440 minutes
*******************************************************************************
Center geodetic : 0.00000000,0.00000000,0.0000000 {E-lon(deg),Lat(deg),Alt(km)}
Center cylindric: 0.00000000,0.00000000,0.0000000 {E-lon(deg),Dxy(km),Dz(km)}
Center radii    : (undefined)
Output units    : KM-S
Output type     : GEOMETRIC cartesian states
Output format   : 2 (position and velocity)
Reference frame : ICRF/J2000.0
Coordinate systm: Ecliptic and Mean Equinox of Reference Epoch
*******************************************************************************
JDTDB
   X     Y     Z
   VX    VY    VZ
*******************************************************************************
$$SOE
2458088.500000000 = A.D. 2017-Dec-01 00:00:00.0000 TDB
 X = 2.962290237798805E+05 Y = 8.933135568594927E+05 Z =-1.862353462813556E+04
 VX=-9.697674894574029E-03 VY= 9.055119172640794E-03 VZ= 2.323585079015175E-04
2458089.500000000 = A.D. 2017-Dec-02 00:00:00.0000 TDB
 X = 2.953905510063761E+05 Y = 8.940954331161901E+05 Z =-1.860344227345183E+04
 VX=-9.711409762306828E-03 VY= 9.043870924106768E-03 VZ= 2.327424977207775E-04
2458090.500000000 = A.D. 2017-Dec-03 00:00:00.0000 TDB
```

The object properties, such as mass, can be found in the "Object Data Page":

Object Data Page

```
Revised : Jul 31, 2013                  Sun                                 10

PHYSICAL PROPERTIES (revised Jan 16, 2014):
 GM (10^11 km^3/s^2)    = 1.32712440041938  Mass (10^30 kg)    ~ 1.988544
 Radius (photosphere)   = 6.963(10^5) km  Angular diam at 1 AU  = 1919.3"
 Solar Radius (IAU)     = 6.955(10^5) km  Mean density          = 1.408 g/cm^3
 Surface gravity        =  274.0 m/s^2    Moment of inertia     = 0.059
 Escape velocity        =  617.7 km/s     Adopted sidereal per  = 25.38 d
 Pole (RA,DEC in deg.)  =  286.13,63.87   Obliquity to ecliptic = 7 deg 15'
 Solar constant (1 AU)  = 1367.6 W/m^2    Solar lumin.(erg/s)   =  3.846(10^33)
 Mass-energy conv rate  = 4.3(10^12 gm/s) Effective temp (K)    =  5778
 Surf. temp (photosphr)= 6600 K (bottom) Surf. temp (photosphr)=  4400 K (top)
 Photospheric depth     = ~400 km         Chromospheric depth   = ~2500 km
 Sunspot cycle          = 11.4 yr         Cycle 22 sunspot min. =  1991 A.D.

 Motn. rel to nrby strs= apex : RA=271 deg; DEC=+30 deg
                         speed: 19.4 km/s = 0.0112 AU/day
 Motn. rel to 2.73K BB = apex : l=264.7+-0.8; b=48.2+-0.5
                         speed: 369 +-11 km/s
```

Hints:

- To allow an intuitive interpretation of the results, a Java program called `VRL-Solar-System-Viewer` has been published on the webpage. It allows the visulization of your simulated solar system. The program requires a recent Java installation (Version > 1.8.70). To start the program, either double-click on the .jar-file or run the program via `java -jar VRL-Solar-System-Viewer.jar`. The source code is also available on GitHub.

- Stick to the order of the planetary objects as specified above. Otherwise, results cannot be interpreted by the `VRL-Solar-System-Viewer`. Use the time slider at the bottom of the program window to visualize planetary movements.

Tasks/Questions:

(a) Setup the right-hand side for the N-Body system with the 13 bodies listed above and simulate the time interval $[t_0 = \mathtt{0}, t_n = \mathtt{3.1558149E7\ sec}]$ with $TOL = \mathtt{1e\text{-}4}$, $h_{min} = \mathtt{0.01}$ and $h_{max} = \mathtt{1e6}$. Use the "VectorTrajectoryToFile" component to save your results as text file. To visualize your results, use the `VRL-Solar-System-Viewer.jar`. Compare the location of the earth at t_0 and t_n. Since the specified time-span is exactly one earth orbit, this is a good estimation for the global error of your simulation.

(b) Change the simulation interval to $[t_0 = \mathtt{0}, t_n = \mathtt{1\ year}]$ and compare your results with the numbers published by NASA. Specify the global error.

(c) Simulate the time intervals $[t_0 = \texttt{0}, t_n = \texttt{10 years}]$ and $[t_0 = \texttt{0}, t_n = \texttt{100 years}]$. Please note that on older machines the computation can take a while. It is adviced to filter the solution trajectory, e.g., only add every n-th value to the solution. For the last interval, it is allowed to increase the TOL value to `1e-2` if your computer is too slow. If you do so, specify the TOL value that has been used for the computation.

Exercise 2 (3 Points)
Design and simulate your own solar system with at least 3 bodies.

Exercise 3 (5 Bonus Points)
Write a VRL component that reads the Ephemeris data from the HORIZONS web interface. Plot the difference between the computed trajectory by your ODE solver for planet earth and the trajectories published by NASA. The result is a sufficient estimate for the global error.

Remark: Send your implemented source code as VRL-Studio project (.vrlp file) and the answers to the questions as plain text in an email. Append the pdfs produced with the TrajectoryPlotter to the email.
Send your solution to `practical.sim1@gcsc.uni-frankfurt.de` until Monday, 11.12.2017, 10h.