A Non-Standard Finite Element Method based on Boundary Integral Operators

by
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Abstract

We present and analyze a new non-standard finite element method based on element-local boundary integral operators that permits polyhedral element shapes as well as meshes with hanging nodes. The method employs elementwise PDE-harmonic trial functions and can thus be interpreted as a local Trefftz method. The construction principle requires the explicit knowledge of the fundamental solution of the partial differential operator, but only locally, i.e., in every polyhedral element. This allows us to solve PDEs with elementwise constant coefficients.

In this talk we mainly consider the diffusion equation as a model problem, but the method can be generalized to convection-diffusion-reaction problems and to systems of PDEs like the linear elasticity system and the time-harmonic Maxwell equations with elementwise constant coefficients. We provide a rigorous H^1- and L_2-error analysis of the method under quite general assumptions on the geometric properties of the polyhedral elements. Numerical results confirm our theoretical estimates. Finally, we discuss solution techniques for the systems of algebraic equations, and we consider the application of this method to other PDEs such as mentioned above.

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