G-CSC Kolloquium

Some features and challenges of the Navier-Stokes-alpha-beta equation

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Abstract

The Navier-Stokes-alpha-beta equation regularizes the Navier–Stokes equation by the addition of dispersive and dissipative terms. The dispersive term is proportional to the divergence of the corotational rate of the symmetric part of the velocity gradient. The dissipative term is proportional to the bi-Laplacian of the velocity. The coefficients of these terms involve factors alpha and beta, respectively, both having dimensions of length. Calculating the energy spectrum for an assembly of stretched spiral vortices reveals an inertial range where Kolmogorov's -5/3 law holds and shows that choosing beta less than alpha yields a better approximation of the inertial range of the Navier–Stokes equation. Direct numerical simulations of three-dimensional periodic turbulent flow confirm this and also show that vorticity structures behave more realistically when beta is less than alpha. However, the simulations indicate that optimal choices of alpha and beta are resolution dependent. This suggests the possibility of developing multigrid methods that capitalize on resolution dependence by using the Navier–Stokes-alpha-beta equation at coarse grid levels, with different choices of alpha and beta at each level, to accelerate convergence to solutions of the Navier–Stokes equation at the finest grid level. Results obtained from a two-dimensional spectral multigrid algorithm of this type show promise.