Regime switching processes are considered to be excellent models of electricity prices and natural gas spot prices. Regime switching processes have also found application in long term insurance guarantees. We analyze a number of techniques for pricing American options under a regime switching stochastic process, using PDE (partial differential equation) methods. The techniques analyzed include both explicit and implicit discretizations with the focus being on methods which are unconditionally stable. In the case of implicit methods we also compare a number of iterative procedures for solving the associated non-linear algebraic equations. Numerical tests indicate that a fixed point policy iteration, coupled with a direct control formulation, is a reliable general purpose method. Finally we remark that we formulate the American problem as an abstract optimal control problem, hence our results are applicable to more general problems as well.