Goethe-Center for Scientific Computing (G-CSC)
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Modelling and Simulation I

(Practical SIM1, WS 2016/17)

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Exercise sheet 0 (warmup) (Due: Mo., 31.10.2016, 10h)

Notation: Spatial components are labeled $x_1, x_2, x_3, ..., x_n$ in the n-dimensional space $\mathbb{R}^n$. Vectors are denoted by bold symbols,

$$\mathbf{x} := \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}.$$

Exercise 1 (Gradient, 3 P)
Compute the gradient $\nabla f(\mathbf{x})$ for the following functions:

(i) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(\mathbf{x}) = 3x_1^4 + 2x_1x_2 - 6x_1x_2^2$,

(ii) $f : \mathbb{R}^n \to \mathbb{R}$, $f(\mathbf{x}) = e^{-\|\mathbf{x}\|^2}$,

with the Euclidean norm $\|\mathbf{x}\| = \sqrt{x_1^2 + \ldots + x_n^2}$.

Exercise 2 (Divergence, 3 P)
Compute the divergence $\nabla \cdot \mathbf{f}(\mathbf{x})$ for the following vector fields:

(i) $\mathbf{f} : \mathbb{R}^2 \to \mathbb{R}^2$, $\mathbf{f}(\mathbf{x}) = \begin{pmatrix} x_1^2 + x_2 \\ x_2^3 + x_1x_2 \end{pmatrix}$,

(ii) $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n$, $\mathbf{f}(\mathbf{x}) = \mathbf{x}$.

Exercise 3 (Jacobian, 4 P)
Compute the Jacobian $\mathbf{J}(\mathbf{x})$ for the following function:

$$\mathbf{f} : \mathbb{R}^3 \to \mathbb{R}^2,\ \mathbf{f}(\mathbf{x}) = \begin{pmatrix} x_1 + x_2^2 + x_3^3 + x_1x_2x_3 \\ e^{2x_1+4x_2} \end{pmatrix}.$$

Exercise 4 (Laplace, 6 P)
Compute the Laplacian $\Delta f(\mathbf{x})$ of the following functions:

(i) $f : \mathbb{R}^2 \to \mathbb{R},\ \mathbf{x} \mapsto f(\mathbf{x}) = \sin(\lambda \cdot x_1) + \sin(\lambda \cdot x_2)$,

(ii) $f : \mathbb{R}^n \to \mathbb{R},\ \mathbf{x} \mapsto f(\mathbf{x}) = \|\mathbf{x}\|^2$,

(iii) $f : \mathbb{R}^3 \to \mathbb{R},\ \mathbf{x} \mapsto f(\mathbf{x}) = \frac{1}{\|\mathbf{x}\|},\ \mathbf{x} \neq \mathbf{0}$,

with Euclidean norm $\|\mathbf{x}\| = \sqrt{x_1^2 + \ldots + x_n^2}$.

Exercise 5 (Calculation rules for the gradient, 4 P)
Consider two partially differentiable functions $f, g : \mathbb{R}^n \to \mathbb{R}$. Show:

(i) $\nabla(fg) = g\nabla f + f\nabla g$,

(ii) $\nabla \cdot (\nabla f) = \Delta f$.

Hint: You may assume that the chain rule for functions of one variable, $f, g : \mathbb{R} \to \mathbb{R}$, $\partial_i(f \cdot g) = f \cdot \partial_i g + g \cdot \partial_i f$, is known and valid.

Remark: Send your answers to the questions as plain text / pdf / scanned pages via email to `practical.sim1@gcsc.uni-frankfurt.de` until Monday, 31.10.2016, 10h.