G-CSC Kolloquium

Janosch Rieger, Universität Frankfurt

Uncertainty Quantification and Partial Differential Inclusions

Abstract

Uncertainty quantification is an area of increasing practical importance.In some applications, it is desirable to understand the distribution of thesolution of an operator equation or a partial differential equation provided that the right-hand side is a random variable with known distribution.This type of uncertainty is currently an object of intense research.

A very different type of problem arises if only bounds and no distributions are known for the right-hand sides. The elliptic partial differential inclusion
\begin{equation} \label{elliptic}
Au \in F(u)\ \text{in}\ \Omega, \quad u=0\ \text{on}\ \partial\Omega \tag{$*$}
\end{equation}
considered in this talk models deterministic uncertainty and constrained control problems. We currently try to develop the necessary analytical background and numerical methods for an efficient approximation of the set of all solutions of \eqref{elliptic}.

First experiments in the linear elliptic case show that it is difficult to obtain good results by discretizing the multivalued right-hand side F. It is much better to project inclusion \eqref{elliptic} to some finite-dimensional space and approximate the solution set of the resulting algebraic inclusion.

The semi-linear elliptic case is much more involved. Set-valued Nemytskii operators have to be considered for the projection of \eqref{elliptic} to a finite element space. In order to guarantee uniform convergence of the Galerkin solution sets, the so-called \emph{relaxed one-sided Lipschitz property} is imposed on the right-hand side F, which is a generalization of the classical OSL property. Under this assumption, it is also possible to discretize and approximate the unknown Galerkin solution sets.