Eulerian and Semi-Lagrangian Methods for Convection-Diffusion for Differential Forms

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Abstract: We consider generalized linear transient convection-diffusion problems for differential forms on bounded domains in $\mathbb{R}^n$. While the stable discretization of scalar convection-diffusion has attracted immense attention in numerical analysis, there is little research for the other cases, e.g. vectorial convection-diffusion problems in magnetohydrodynamics.

By now it is widely appreciated that thinking in terms of co-ordinate free differential forms offers considerable benefits as regards the construction of structure preserving spatial discretizations. The so-called discrete exterior calculus, or, equivalently, the mimetic finite difference approach, or discrete Hodge-operators have shed new light on existing discretizations and paved the way for new numerical methods. The notion of differential forms enables to accentuate the main ideas of discretization methods in hiding technical details in a unifying notation.

In the case of convection problems it is the notion of Lie-derivatives with respect to prescribed smooth vector fields that allows for such unifying treatment. We will present both new Eulerian and semi-Lagrangian approaches to the discretization of the Lie derivatives in the context of a Galerkin approximation based on discrete differential forms.